Detection of High Energy Neutrinos produced by the Sun

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1 Introduction

The motivation behind this thesis became a manifesto to the fact that the Sun is the nearest source of high energy neutrinos. Thus, the major aim of this thesis is to determine the number of these neutrinos detected at IceCube. There are two processes explained in detail, the first being the interaction between cosmic rays and the solar atmosphere, the second being dark matter annihilations. The flux is analytically determined like in Refs. [1] and [2] and also oscillations are taken into account.

In terms of cosmic rays entering solar matter, secondary particles come into existence whose decay results in neutrino production.

Current dark matter theories assert that high energy neutrinos could also be produced through dark matter annihilations in the Sun. Yet the nature of dark matter is still unknown but its occurrence is supposed to be about five times higher than the familiar kind.

In this thesis the outline is as follows: Chapter 2 gives an overview of the IceCube neutrino detector and its operating mechanisms. In chapter 3 the two neutrino production processes are discussed, where in the first case, calculation techniques are presented that explain how cosmic ray particles react with solar matter and how neutrinos are produced due to this. An essential part here is obtaining the resulting neutrino flux at the Earth. Subsequently the theory of dark matter is taken into account and it is described how neutrino production works in this case. Chapter 4 is concerned with neutrino oscillations in a three flavor case, giving all probabilities and plots of resulting fluxes. Finally, the event rates expected at IceCube are analytically calculated in chapter 5 and chapter 6 summarizes all the results and compares them with the results a former calculation like in [3].
2 IceCube Neutrino Observatory

2.1 General Information

Located deep under the South Pole’s surface, construction of the IceCube Neutrino Detector was successfully completed in December of 2010. It is supposed to detect high energy neutrinos from a series of possible sources such as radioactive decays, cosmic rays and supernovae. Due to the low neutrino-nucleon cross section, it is necessary that neutrino detectors are of a large volume in order to achieve a non-vanishing event rate per year. For that reason, IceCube encompasses a geometric volume of one cubic kilometer of ice. It is composed of 86 strings containing 5,160 optical sensors, so called digital optical modules (DOMs). The absence of daylight from depths of 1,450 m to 2,450 m and the detection medium being due to high pressure optically ultra transparent, the DOMs are able to detect flashes of blue light emitted by secondary particles.

2.2 Detection Methods

Neutrinos are leptons that only gravitationally and weakly interact with others since they are electrically neutral, making this good and bad. On the one hand, the low interaction rate with matter allows to determine accurately the neutrino’s origin, but on the other hand, it makes them hard to detect. There are two possible ways for neutrinos to interact with a nucleus. The first one is the neutral current (NC) [4]

\[ \nu_\ell + N \rightarrow Z^0 \rightarrow \nu_\ell + X, \]  

(2.1)

and the second one is the charged current (CC) [4]

\[ \nu_\ell + N \rightarrow W^\pm \rightarrow \ell + X. \]  

(2.2)

In the above equations \( \nu_\ell \) describes the participating neutrino in the interaction process, \( N \) the nucleus, \( Z^0 \) and \( W^\pm \) are the gauge bosons. The character \( X \) identifies a hadronic cascade and \( \ell \) represents a lepton of the same flavor as the incoming neutrino.

The neutrino flux is measured indirectly by Cherenkov light, which the generated charged particles emit when they travel faster than the speed of light in a dielectric medium, like ice. These particles polarise with their electromagnetic field the atoms that are near to the particles’ trace. As soon as the atoms leave their excited state, they emit photons that destructively interfere when the particles’ speed is lower than the speed of light. In the other case, when they are faster than light, the photons develop constructive interferences and wave fronts establish. The result is a light cone, whose angle depends on the particles’ speed and the refraction index. In ice, the angle reaches a maximal value of \( \theta \approx 40^\circ \) for particles traveling with almost the speed of light. This light can finally be detected by the DOMs.

Therefore, it makes sense to concentrate on the CC-process, because of the appearance of charged particles.
3 The Sun as a high energy neutrino emitter

In this paper, the Sun is examined as a high energy neutrino source where the following neutrino production processes are considered. At first the focus is put on inelastic hadronic interactions between cosmic rays and the particles of the solar atmosphere before concentrating on WIMP annihilation processes.

If not otherwise stated, this section refers to the work of [1] and [2].

3.1 Proton-proton interaction

Incidentally cosmic rays primarily cause a proton-proton interaction, which leads to the production of secondary particles. Equations (3.1) thru (3.3) show how interaction processes are linked to pion production [4]. Later, charged pions decay into muon and electron neutrinos.

\[
p p \rightarrow \begin{cases} 
  p p \pi^0, \text{ fraction } 2/3 \\
  p n \pi^+, \text{ fraction } 1/3,
\end{cases}
\]  

(3.1)

\[
\pi^+ \rightarrow \mu^+ \nu_\mu \rightarrow e^+ \nu_e \bar{\nu}_\mu \nu_\mu
\]

(3.2)

\[
\pi^- \rightarrow \mu^- \bar{\nu}_\mu \rightarrow e^- \bar{\nu}_e \nu_\mu \bar{\nu}_\mu.
\]

(3.3)

Negative charged pions arise if the incoming protons in Eq. (3.1) are replaced by neutrons. These neutrons rather interact with solar matter instead of decaying beforehand and also produce neutrinos.

The appearance of nuclear collisions can be treated as nucleon-nucleon interactions because of their insignificant binding energies. A detailed simulation about the production of secondary particles and their decay into neutrinos is provided by the Lund Monte Carlo programs PYTHIA and JETSET [5].

Cosmic rays

Nuclei claim the largest part of the primary cosmic ray spectrum, the other part, represented by only 2%, is made up of electrons [6]. The nuclei flux can be written as [2]

\[
\phi_N(E) \left[ \frac{\text{nuclei}}{\text{cm}^2 \text{ s} \text{ sr} \text{ GeV} / \text{A}} \right] = \begin{cases} 
  1.7E^{-2.7}, & E \leq 5 \times 10^6 \text{GeV} \\
  174E^{-3}, & E > 5 \times 10^6 \text{GeV}.
\end{cases}
\]  

(3.4)

In general the cosmic ray flux is described by a broken power law with knee and ankle. The ankle in the flux describes it at energies exceeding \( \sim 10^9 \text{GeV} \), which in this papers’ context are neglected and therefore not mentioned. The nucleon part of the spectrum consists mostly of protons with a smaller fraction arranged by \( \alpha \)-particles and heavier nuclei.
Solar atmosphere

The outer atmosphere of the Sun is made up of hydrogen and a smaller fraction of helium and its density profile is given by

\[ \rho(h) = \rho_0 e^{-h/h_0}, \]

where \( h > 0 \) and \( h < 0 \) gives information about locations above and below the solar radius \( R_\odot = 6.69 \times 10^5 \text{km} \). The values for \( \rho_0 \) and \( h_0 \) can be taken from table 1.

<table>
<thead>
<tr>
<th>height ( h ) [km]</th>
<th>( \rho_0[\text{g}\cdot\text{cm}^{-3}] )</th>
<th>( h_0[\text{km}] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( h &gt; 0 )</td>
<td>( 3.68 \times 10^{-7} )</td>
<td>115</td>
</tr>
<tr>
<td>( -2000 \leq h &lt; 0 )</td>
<td>( 3.68 \times 10^{-7} )</td>
<td>622</td>
</tr>
<tr>
<td>( h &lt; -2000 )</td>
<td>( 45.3 \times 10^{-7} )</td>
<td>2835</td>
</tr>
</tbody>
</table>

Table 1: The values of the parameters \( \rho_0 \) and \( h_0 \) in equation (3.5). Table adopted from [2].

Cosmic rays may enter deep into the Sun, so that neutrinos may also be produced in its center. In this case, secondary particles have to pass through the Sun’s high density profile until they are able to reach the Earth. Consequently, the impact parameter \( b \) indicates the place where the collision takes place in the Sun, so \( R_\odot > b > 0 \). Moreover, in this calculation, \( b \) is treated with fixed values (\( b = 0, b = 2R_\odot, b = R_\odot \)) in order to get a less complex Monte Carlo simulation.

Primary particle propagation

For the purpose of describing a particle’s propagation through the solar medium a set of transport or cascade equations is necessary. The transport equations for nucleons, mesons, unstable baryons and leptons are generally coupled but a few assumptions that do not significantly affect the results have been made. As a first assumption, the small contribution to the nuclear flux through the interaction of unstable hadrons with nuclei from solar matter is not taken into account, while nucleon absorption and regeneration in inelastic collisions indeed contribute to this calculation. Taking \( n_A(h) \) as the number density of nuclei \( A \) at height \( h \) and the inclusive inelastic cross section \( \sigma_{NA} \) for collisions of cosmic rays with nuclei \( A \), the following equation describes the interaction thickness is determined by

\[ \lambda_N(E) = \frac{\rho(h)}{\sum_N \sigma_{NA}(E)n_A(h)}. \]

The so-called slant depth \( X \) is obtained by integrating over the Sun’s atmosphere density along the incoming nucleons trajectory.
\[ X(h) = \int_\ell^\infty d\ell' \rho(\ell'). \] (3.7)

The flux of nucleons at depth \( X \) can now be written as

\[ \frac{d\phi_N}{dX} = -\frac{\phi_N}{\lambda_N} + S(NA \rightarrow NY), \] (3.8)

where \( S(NA \rightarrow NY) \) describes the regeneration function of high energy nucleon-nucleon interaction in solar matter. As a consequence, the initial cosmic ray proton flux \( \phi_0 \) with a spectrum according to Eq. (3.1) propagates like Eq. (3.8) without regenerating, the flux becomes

\[ \phi(h) = \phi_0 \exp\left(-\frac{X}{\lambda_N}\right), \] (3.9)

while propagating through the Sun, when regeneration effects are neglected. The following uniformly distributed random number \( R = \frac{\phi(h)}{\phi_0} \in (0, 1) \) is defined in the equation (3.10) which gives information about the height \( h \), where the primary interaction of cosmic rays with solar matter is settled

\[ \ln(R) = -\int_{\infty}^{h} dh \sigma n(h). \] (3.10)

\textsc{Pythia} then calculates to great detail proton-proton interactions and gives a final state of particles. Secondary particles that are produced in the interaction process (see Eq. 3.1) either decay or produce cascades themselves when they also interact while secondary nucleons of less then 30\% of primary flux’ energy are neglected in further calculations as their flux is too small. Then, the secondary nucleons that do contribute to the primary flux, because of their high energy, are allowed to evoke a third flux. Thus, the interaction at a certain height is given by

\[ \ln(R') = -\int_{H}^{h} dh \sigma n(h); \] (3.11)

where \( H \) is the height where the secondary nucleon that caused this third flux has been produced and \( R' = \frac{\phi(h)}{\phi_0(H)} \). This pattern of calculation is continued as long as the energy of the nucleon, that has been produced in earlier cascades, is higher than 30\% of the primary cosmic ray energy, otherwise it is stopped.

Whether secondary particles decay or interact is determined by the comparison of interaction length \( L_i \) and decay length \( L_d \):
\[ L_d = -c\beta\gamma\tau \ln R_1, \quad (3.12) \]
\[ L_i = \frac{H + h_0}{e^{H/h_0} - \frac{\lambda_N \ln R_2}{X_0}}, \quad (3.13) \]

where \( \beta = v/c, \gamma = E/mc^2 \) and \( \tau \) is the mean lifetime. While daughter particle momenta also have been taken into account for particle decays, only the most energetic, so called leading particles, have been considered for secondary interactions that cause the third flux. This is because only these contribute to the high energy neutrino flux. The fact that third fluxes mainly take place deeper in the solar matter, where density increases the produced particle tends to interact instead of decaying, which make them less important. Then, this procedure is iterated for not yet decayed particles with energies higher or equal 100 GeV. Eventually, in a last step, the neutrino energy spectrum results in the number of produced neutrinos and weighted with the primary proton energy.

Muons propagate

As muons are leptons the strong force is not affecting them. Therefore they rather continuously lose fractions of their energy in several scattering processes and the energy loss formula is represented by

\[ \frac{dE}{dh} = \alpha \rho + \beta \rho E, \quad (3.14) \]

with the parameters \( \alpha = 2.5 \times 10^{-3} \text{ GeV cm}^2 \text{ g}^{-1} \) and \( \beta = 4.0 \times 10^{-6} \text{ cm}^{-2} \text{ g} \). The small but numerous stepwise energy losses for muons does not allow to accomplish the calculation as described above. Therefore it is separated in such a way that \( dE/dh \) is nearly constant \( (\Delta E/E \approx 10\%) \). Likewise, the above calculation recipe can now be applied for those muons whose energy is greater than a 100 GeV and have not yet decayed. The neutrino flux is then analogously determined.

Neutrino flux

Considering the fact that neutrinos weakly interact with solar matter, their flux as calculated in paragraphs above is going to be damped. The interaction is differentiated between charged (CC) and neutral (NC) currents. While in charged currents the neutrino completely disappears, it loses some of its energy in neutral currents. Analogous to the transport of the primary particles, the neutrino flux \( \phi_\nu \) is determined by

\[ \frac{d\phi_\nu}{dX} = -\phi_\nu \frac{1}{\lambda_\nu} + S(\nu A \to \nu Y). \quad (3.15) \]
In correspondence to Eq. (3.8) $\phi_\nu$ identifies the absorption and implies energy losses through CC- and NC-interactions and $S(\nu A \rightarrow \nu Y)$ is the regeneration function, which can be expressed by

$$S(\nu A \rightarrow \nu Y) = \frac{\phi_\nu(E)}{\lambda_\nu(E)} Z.$$  

(3.16)

The function of the $Z$-moment implies an integral that includes regeneration. $Z$ has been calculated with PYTHIA and is graphically presented in Fig. 1. Approximately, $Z$ can be used for muon and electron neutrinos, because their cross sections as well as their spectral form right after being produced is nearly the same. Solving Eq. (3.15) the damped neutrino flux yields

$$\phi_\nu(r) = \phi_{\nu,0} \exp \left[ \frac{(Z - 1)X(r)}{\lambda_\nu(E_\nu)} \right],$$  

(3.17)

where $X(r)$ is the distance the neutrino has passed in solar matter. The flux $\phi(E)$ has been treated as a function of the impact parameter $b$ and then fitted to the flux integrated over the Sun. Mostly, neutrino production takes place in a tiny outer fraction of the Sun. Therefore, the exponential function in Eq. (3.17) can be replaced by an overall attenuation function for the Sun, that is represented in terms of the parameter $b$

$$A(E_\nu, b) = \exp \left[ \frac{\sigma(E_\nu) (Z - 1) X(b)}{m_N} \right],$$  

(3.18)

where $X(b)$ stands for the Sun’s effective thickness and $m_N$ is the nucleon mass. Eq. (3.18) has been numerically calculated and Fig. 3.1 demonstrates this function graphically.

The resulting flux is determined by interpolating low-energy and high-energy asymptotic solutions that result from cascade equations (see Eq. 3.8). This has been done by taking into account power law primary spectra, scale-invariant cross sections and non-scaling effects. First assumption here is to treat the flux of neutrinos as $\phi_i(E) = E^{-\beta_i} \phi_i$, where $\beta_i$ characterizes low-energy and high-energy regimes. Since the neutrino flux is a result from the cosmic ray flux, the primary spectrum of cosmic rays $\phi_N = E^{-\gamma-1}$, with $\gamma$ as a constant spectral index, as well as its primary spectrum with a knee (Eq. 3.4) have been included to this calculation. Further assumptions affecting meson regeneration and decay $Z$-moments amongst others have been made. Then, taking into account the factor $R^2_\odot/D^2$, where $D$ is the distance between Sun and Earth, it has been interpolated between the different impact parameters $b$ and folded with the attenuation factor. The integral over the solar disc results in the total neutrino flux at Earth:

$$\phi_\nu(E) = \frac{1}{1 + AE} \cdot \begin{cases} N_0 E^{-\gamma-1}, & E \leq E_0 \\ N'_0 E^{-\gamma'-1}, & E > E_0 \end{cases},$$  

(3.19)
with the parameter:

\[
\begin{array}{cccccc}
\nu_\mu + \bar{\nu}_\mu & 1.3 \times 10^{-5} & 1.98 & 8.5 \times 10^6 & 3.0 \times 10^6 & 2.38 & 5.1 \times 10^{-3} \\
\nu_e + \bar{\nu}_e & 7.4 \times 10^{-6} & 2.03 & 8.5 \times 10^6 & 1.2 \times 10^6 & 2.33 & 5.0 \times 10^{-3} \\
\end{array}
\]

Table 2: Flux parameter as a result of the above described calculation. Table adopted from ref. [2]

Figure 1: \( Z_{\nu \nu} \) moment as a function of energy. It has been calculated with the Monte Carlo program PYTHIA, ref. [2]

Figure 2: The graphical presentation of the attenuation factor, ref. [2]
3.2 WIMPs

The problem of dark matter arises when rotating curves of galaxies are analyzed. The Newtonian dynamics describe the tangential velocity \( v(r) \) of hydrogen gas as \( v(r) \propto r^{-1/2} \), where \( r \) is the distance from the galactic center. On the contrary it turns out that the observed velocity is of a constant value at great distances \([7]\). This leads to the assumption of dark matter, so that the relation \( M(r) \propto r \), where \( M(r) \) is the total mass, is fulfilled. Several models have been introduced in order to solve this problem and high attention is paid to hypothesized weakly interacting massive particles (WIMPs). Since the additional mass is not visible to electromagnetic wavelength, the WIMPs are expected to merely interact through gravitation and weak force.

MSSM

Supersymmetry (SUSY) describes a symmetry between elementary particles that correlates integer spin particles to half integer spin particles \([8]\). If two particles transform into one another through SUSY-transformation they are called superpartners. Both would only differ in spin and therefore have the same mass. No supersymmetric particles have yet been found, so it might be a broken symmetry. The WIMP annihilation branching ratios depend on the considered model and the minimal supersymmetric extension to the standard model (MSSM) is taken into account \([9]\). Table 3 shows the normal particles as given in the standard model in combination with their supersymmetric partners. In the MSSM there are two charged and three neutral Higgs bosons required whose superpartners mix up with the electroweak gauge bosons. The MSSM also makes use of the so called \( R \)-parity, which is a conserved quantity

\[
R = (-1)^{3(B-L)+2S}.
\]

(3.20)

Here \( B \) denotes the baryon number, \( L \) is the lepton number and \( S \) is the spin. This equation allocates each and every standard model particle \( R = +1 \) and all SUSY particles \( R = -1 \). The lightest supersymmetric particle would be stable and mostly the \( \tilde{\chi}_1^0 \) neutralino, which is a merely weakly interacting Majorana particle (i.e. particle and antiparticle at the same time). This neutralino may be a perfect dark matter WIMP whose mass is supposed to be greater than 47 GeV, as experiments in the LEP-accelerator have found out \([10]\). In the following the notation \( \chi \equiv \tilde{\chi}_1^0 \) will be used.
Table 3: Representation of particles and fields in the standard model, together with their partners in the MSSM. Table adapted from Ref. [11]

**WIMP annihilations**

WIMPs lose a fraction of their velocity in scattering processes. Hence they can be trapped by dense objects like the Earth or the Sun when they experience repeated scattering on nucleons over cosmological timescales [9]. An approximation of the capture rate of a dark matter particle in the Sun is given by [12]:

\[
C_\odot \approx 3.4 \times 10^{-20} \text{s}^{-1} \left( \frac{\rho_{DM}}{0.3 \text{ GeV/cm}^3} \right) \left( \frac{270 \text{ km/s}}{\bar{v}_{DM}} \right)^3 \times \left( \frac{\sigma_{SD} + \sigma_{SI} + 0.07\sigma_{SI}^{He}}{10^{-6} \text{ pb}} \right) \left( \frac{100 \text{ GeV}}{m_\chi} \right)^2 \tag{3.21}
\]

In Eq. (3.21) $\rho_{DM}$ is the local halo density, $\bar{v}_{DM}$ displays its velocity dispersion, $m_\chi$ stands for the WIMP’s mass. Almost every capture event occurs on hydrogen nuclei both in a spin-dependent and spin-independent fashion. Helium and heavier elements only slightly contribute to this process. When using the conventional halo model, the following values are given: $\rho_{DM} = 0.3$ GeV/cm$^{-3}$ and $\bar{v}_{DM} = 270$ km/s.

The balance between capture events, annihilation processes and thermal evaporation from the Sun for the number $N$ of WIMPs is given by the differential equation

\[
\frac{dN}{dt} = C_\odot - C_A N^2 - C_E N. \tag{3.22}
\]
In ref. [13] the evaporation rate $C_E$ is claimed to be negligible when WIMP masses exceed 5 GeV. By solving the above equation and taking the Sun’s age of about $T_{\text{sun}} \approx 5 \times 10^9$ years the capture and annihilation rates are balanced out. The annihilation rate $\Gamma_A = \frac{1}{2} C_A N^2$ then forms as follows:

$$\Gamma_A(t) = \frac{1}{2} C_\odot \tanh^2(t\sqrt{C_\odot C_A}).$$ \hspace{1cm} (3.23)

Considering timescales $t \gg 1/\sqrt{C_\odot C_A}$, which is given for the Sun’s current age, this equation can be approximated to

$$\Gamma_A(t) = \frac{1}{2} C_\odot. \hspace{1cm} (3.24)$$

The area where dark matter (mass: $m_\chi$) annihilations take place is confined to about $\sim 0.01 R_\odot \sqrt{100 \text{GeV}/m_\chi}$ [14]. Since the density around the Sun’s core is $\rho = 140 \text{ g cm}^{-3}$, only neutrinos are able to leave this area and find a way outside the Sun. Although WIMPs do not annihilate directly into $\nu\bar{\nu}$ in the MSSM, the following channels produce muon neutrinos [15]:

$$\chi\chi \rightarrow c\bar{c}, \ b\bar{b}, \ t\bar{t}, \ \tau^+\tau^-, \ W^+W^-, \ Z^0Z^0, \ Z^0H^0_1, \ Z^0H^0_2, \ H^0_1H^0_3, \ H^0_2H^0_3, \ \text{and} \ H^\pm W^\mp. \hspace{1cm} (3.25)$$

Detection at IceCube concentrates on a particle’s energy and direction, so resulting neutrino spectra are of great interest. The channel $\chi\chi \rightarrow b \bar{b}$ creates the softest possible spectrum and the channel $\chi\chi \rightarrow W^+W^-$ the hardest one.
4 Neutrino Oscillation

Due to the non-vanishing mass eigenvalues of neutrinos their flavor is expected to oscillate on their way from the Sun to Earth. The probability of a neutrino with flavor $\alpha$ arriving at Earth as a flavor $\beta$ neutrino is given by \[ P_{\nu\alpha \rightarrow \nu\beta} = \delta_{\alpha\beta} - \left| \sum_i U_{\alpha i} U_{\beta i}^* e^{-iE_it} \right|^2, \] (4.1)

where $U$ is the three-dimensional mixing matrix, also known as PNMS-matrix:

\[
U = \begin{pmatrix}
\cos(\Theta_{12}) & \sin(\Theta_{12}) & 0 \\
-\sin(\Theta_{12}) & \cos(\Theta_{12}) & 0 \\
0 & 0 & 1
\end{pmatrix} \times \begin{pmatrix}
\cos(\Theta_{13}) & 0 & \sin(\Theta_{13}) \\
0 & 1 & 0 \\
-\sin(\Theta_{13}) & 0 & \cos(\Theta_{13})
\end{pmatrix}
\] (4.2)

\[
U = \begin{pmatrix}
c_{12}c_{13} & s_{12} & c_{12}s_{13} \\
-c_{13}s_{12} - s_{13}s_{23} & c_{12}c_{23} & c_{13}s_{23} - c_{23}s_{13}s_{12} \\
c_{13}s_{12}s_{23} - c_{23}s_{13} & -c_{12}s_{23} & s_{12}s_{13}s_{23} + c_{13}c_{23}
\end{pmatrix}
\] (4.3)

with $c_{ij} = \cos(\Theta_{ij})$ and $s_{ij} = \sin(\Theta_{ij})$. In general the mixing matrix also includes a CP-violating phase $\delta$, which leads to the assumption of a sterile neutrino. Here, the sterile neutrino is neglected and $\Theta_{13}$ is assumed to be zero.

The values of the occurring mixing angles and squared mass differences have been experimentally determined according to Tab 4.

| $\Theta_{12} \equiv \Theta_{\text{sun}}$ | $0.54616$ | $\delta m_{21}^2 [10^{-5} \text{eV}^2]$ | $8.1$ |
| $\Theta_{23} \equiv \Theta_{\text{atm}}$ | $\pi/4$ | $|\delta m_{31}^2| [10^{-3} \text{eV}^2]$ | $2.2$ |

Table 4: Parameters for three-flavor neutrino oscillation. The data has been taken from Ref. [17].

Using\[
\alpha = \frac{\delta m_{21}^2}{\delta m_{31}^2} \\
\Delta = \frac{\delta m_{31}^2 L}{4E}
\] (4.4)

where $L = 149.6 \times 10^6$ km is the distance between Sun and Earth, the following probabilities have been used for further calculation:
\[ P_{\nu_e \rightarrow \nu_e} = 1 - \sin^2 (\alpha \Delta) \sin^2 (2\Theta_{\text{sun}}), \]  
(4.5)  
\[ P_{\nu_{\mu} \rightarrow \nu_e} = \cos^2 (\Theta_{\text{atm}}) \sin^2 (\alpha \Delta) \sin^2 (2\Theta_{\text{sun}}), \]  
(4.6)  
\[ P_{\nu_{\mu} \rightarrow \nu_{\mu}} = 1 - P_{\nu_e \rightarrow \nu_e} - P_{\nu_{\mu} \rightarrow \nu_e}, \]  
(4.7)  
\[ P_{\nu_{\mu} \rightarrow \nu_{\mu}} = 1 - \sin^2 (2\Theta_{\text{sun}}) \cos^2 (\Theta_{\text{atm}}) \sin^2 (\alpha \Delta) - \cos^2 (\Theta_{\text{sun}}) \]  
\[ \sin^2 (2\Theta_{\text{atm}}) \left( \sin^2 ((1 - \alpha) \Delta) - \sin^2 (\Theta_{\text{sun}}) \sin^2 (\alpha \Delta) \right) \]  
\[ - \sin^2 (\Delta) \sin^2 (\Theta_{\text{sun}}) \sin^2 (2\Theta_{\text{atm}}), \]  
(4.8)  
\[ P_{\nu_{\mu} \rightarrow \nu_e} = 1 - P_{\nu_{\mu} \rightarrow \nu_{\mu}} - P_{\nu_{\mu} \rightarrow \nu_e}. \]  
(4.9)

According to Eq. (4.1) the probability \( P_{\nu_{\mu} \rightarrow \nu_e} \) is equal to \( P_{\nu_e \rightarrow \nu_{\mu}} \).

Additionally, the neutrino fluxes as mentioned in Eq. (3.19) have been adapted to their oscillations, which has lead to the following corrections:

\[ \tilde{\phi}_e(E) = P_{\nu_e \rightarrow \nu_e} \phi_e(E) + P_{\nu_{\mu} \rightarrow \nu_e} \phi_\mu(E) \]  
(4.10)  
\[ \tilde{\phi}_\mu(E) = P_{\nu_e \rightarrow \nu_\mu} \phi_e(E) + P_{\nu_{\mu} \rightarrow \nu_\mu} \phi_\mu(E) \]  
(4.11)  
\[ \tilde{\phi}_\tau(E) = P_{\nu_e \rightarrow \nu_\tau} \phi_e(E) + P_{\nu_{\mu} \rightarrow \nu_\tau} \phi_\mu(E). \]  
(4.12)

The following figure shows plots of the electron, muon, and tau neutrino flux, respectively. In terms of resulting fluxes, an approximating function has been found for electron and muon neutrinos, that represents the average value of contributing oscillations (see Chapter 5). Since no decay channel produces tau neutrinos, their existence can only be explained through the influence of oscillation. This is why their flux is very small compared to the others and it is therefore neglected in further calculations.
Chapter 4 Neutrino Oscillation

Figure 3: Resulting fluxes of electron, muon and tau neutrinos. Figure (a) and (b) show a blue line for the flux with oscillation (see Eqs. 4.10 thru 4.12), the orange line displays the flux without oscillation (see Eq. 3.19) and the red line indicates the applied approximating function. In case of tau neutrinos (c), only the resulting flux due to oscillation is shown.
5 Event Rates

In order to obtain the total event rate per time unit $\dot{N}_\nu$ of neutrinos exceeding a threshold energy $E_{th}$ the initial flux $\tilde{\phi}_\nu(E)$ due to Eqs. (4.10) thru (4.12) is folded with the probability of detection [4], so that

$$\dot{N}_\nu = \int_{E_{th}}^{\infty} dE \, \tilde{\phi}(E) A_{eff}(E),$$  \hspace{1cm} (5.1)

where the effective area $A_{eff}$ is determined by the probability of detection

$$P_{\nu\ell \rightarrow X} = \rho' N_A \sigma_{CC}(E_\nu) r_{eff}.$$ \hspace{1cm} (5.2)

Here $\rho' = \rho/\rho_{H_2O} \approx 1$, $\sigma_{CC}$ the charged current cross section and $N_A$ is Avogadro’s constant. The parameter $r_{eff}$ denotes the effective range, that approximates the probability of an initial neutrino inducing a cascade and then be detected within IceCube’s range. Hence the effective area depends on the detector’s size because detected cascades must have been induced inside the detection array [4].

Its data is dependent on whether electron- or muon-neutrinos are considered. So, the Figs. 4 and 5 show the individual effective area plot of both neutrino flavors, respectively.

In order to calculate the event rate a piecewise “alternative” function has been found that approximates the oscillation part of the flux. Due to IceCube’s energy resolution particular oscillations cannot be resolved. This is why the in further calculations used arithmetic mean is a good approximation for low energies, $E \lesssim L\delta m^2$, where $L$ is the Sun-Earth distance and $\delta m_{ij}$ the appropriate squared mass difference. See also Fig. 3. In case of higher energies, $E \geq L\delta m^2$, the alternative function is determined by the second order Taylor series expansion of $\sin^2(x)$, since $P_{\nu\ell \rightarrow \nu\ell'} \propto \sin^2(x)$, where $\sin^2(x)$ depends on the neutrino’s energy. Using the approximated transition probabilities of the different neutrino flavors, an analytic solution of the total event rate $\dot{N}_\nu$ is calculated in the following. The integral in Eq. (5.1) has been separated in as many integrals as the given number of bins $n$ and then multiplied with the effective area and in the end, summed up, so that

$$\dot{N}_\nu = \sum_{i=0}^{n-1} \int_{E_i}^{E_{i+1}} dE \, \tilde{\phi}_\nu(E) A_{eff,i}.$$ \hspace{1cm} (5.3)

For each flavor an antiderivative including a hypergeometric function has been found for each piece of the alternative function. The antiderivative is given in the appendix A and the alternative function is presented in corresponding sections.
5.1 Electron neutrinos through cosmic rays

In this section the total event rate for electron neutrinos is calculated. The flux of electron neutrinos produced through cosmic ray interactions with solar matter as well as the influence of oscillation is graphically displayed in Fig. 3(a) and Fig. 4 shows IceCube’s effective area for electron neutrinos.

The alternative function is given by

$$ A_{\nu_e}(E) = \begin{cases} 
0.394 \left( \frac{9.17726 \times 10^6}{E^2} - \frac{2.80741 \times 10^{13}}{E^4} \right) & E < 3.0 \times 10^6 \text{ GeV} \\
0.000013 & E > 3.0 \times 10^6 \text{ GeV} 
\end{cases} $$

$$ \times \begin{cases} 
0.197 & E < 3.0 \times 10^6 \text{ GeV} \\
0.98 & E > 3.0 \times 10^6 \text{ GeV} 
\end{cases} $$

$$ \times \begin{cases} 
0.0053 \left( \frac{8.5 \times 10^{-6} E + 1}{E^{3.5-2}} \right) & E < 1.2 \times 10^6 \text{ GeV} \\
7.4 \times 10^{-6} \left( \frac{8.5 \times 10^{-6} E + 1}{E^{3.5-2}} \right) & E > 1.2 \times 10^6 \text{ GeV} 
\end{cases} $$

Figure 4: The effective area of IceCube over the electron neutrino energy, displayed in equidistant energy bins.
Finally, the event rate of electron neutrinos yields

\[ N_{\nu_e} = 0.46 \, a^{-1}. \]  

(5.5)

### 5.2 Muon neutrinos through cosmic rays

Here the total event rate for muon neutrinos is calculated. Their flux produced by cosmic ray interactions with solar matter as well as the influence of oscillation is graphically displayed in Fig 3(b).

![Figure 5: The effective area of IceCube over the muon neutrino energy, displayed in equidistant energy bins.](image)

With

\[ W_1 = \left( \frac{9.177 \times 10^6}{E^2} - \frac{2.807 \times 10^{13}}{E^4} \right), \]

\[ W_2 = \left( \frac{6.281 \times 10^9}{E^2} - \frac{1.314 \times 10^{19}}{E^4} \right), \]

\[ W_3 = \left( \frac{6.77 \times 10^9}{E^2} - \frac{1.528 \times 10^{19}}{E^4} \right), \]
the alternative function has been calculated as

\[
A_{\nu_\mu} = \begin{cases} 
0.33 & \text{for } E < 8 \times 10^4 \text{ GeV} \\
-0.197W_1 - 0.730W_2 - 0.270W_3 + 1 & \text{for } E > 8 \times 10^4 \text{ GeV}
\end{cases} 
\times \begin{cases} 
\begin{array}{c}
\frac{1.3 \times 10^{-5}}{(8.5 \times 10^{-6} E + 1)^{1.38}} \\
\frac{5.1 \times 10^{-3}}{(8.5 \times 10^{-6} E + 1)^{3.38}}
\end{array} 
& \text{for } E < 3 \times 10^6 \text{ GeV} \\
\begin{array}{c}
0.197 \\
0.394 \left( \frac{9.177 \times 10^6}{E^2} - \frac{2.807 \times 10^{13}}{E^4} \right)
\end{array} 
& \text{for } E > 3 \times 10^6 \text{ GeV}
\end{cases}
\times \begin{cases} 
\begin{array}{c}
\frac{7.4 \times 10^{-6}}{(8.5 \times 10^{-6} E + 1)^{1.38}} \\
\frac{1}{200(8.5 \times 10^{-6} E + 1)^{3.38}}
\end{array} 
& \text{for } E < 1.2 \times 10^6 \text{ GeV} \\
\begin{array}{c}
0.197 \\
0.394 \left( \frac{9.177 \times 10^6}{E^2} - \frac{2.807 \times 10^{13}}{E^4} \right)
\end{array} 
& \text{for } E > 1.2 \times 10^6 \text{ GeV}
\end{cases}
\]

The event rate of muon neutrinos reaches a value of

\[
\dot{N}_{\nu_\mu} = 2.12 \text{ a}^{-1}. 
\]  

### 5.3 Muon neutrinos through WIMP annihilation

This sections considers the muon neutrino flux through WIMP annihilation. According to Eq. (3.25), neutralinos annihilate in several processes, that themselves decay into neutrinos amongst others. For example, in the process $\chi\chi \rightarrow Z^0 Z^0$, the $Z^0$ boson may decays according to $Z^0 \rightarrow \nu\bar{\nu}\gamma\gamma$ [10]. The neutrinos’ energy spectrum is thus not discrete but continuous, where their maximum energy is limited by the annihilated WIMPs mass. Furthermore, in this process the production spectrum of flavor $\ell$ neutrinos is not known and depended on the considered WIMP model. Here, all contributing annihilation channels are weighted with its branching ratio, so that it can be written as [9]

\[
\frac{dN_{\nu_\ell}}{dE_{\nu_\ell}} = \sum_X B(\chi\chi \rightarrow X) \left( \frac{dN_{\nu_\ell}}{dE_{\nu_\ell}} \right)_X, \tag{5.8}
\]

where $X$ denotes the particular channel. The assumption $B(\chi\chi \rightarrow b\bar{b}) = 1$ provides the softest possible spectrum and the most conservative limit. The most optimistic limit is obtained by the hardest possible spectrum, with $B(\chi\chi \rightarrow W^+ W^-) = 1$ [9]. The neutrino to muon conversion rate as well as the muon flux in the detector is directly proportional to the annihilation rate. The muon flux is obtained by [9]

\[
\phi_{\mu}(E_{\mu} \geq E_{th}) = \frac{\Gamma_A}{4\pi L^2} K(E_{th}), \tag{5.9}
\]

where $\Gamma_A$ is the annihilation rate due to Eq. (3.24), $L$ the Sun-Earth distance. The factor $K(E_{th})$, including the incident neutrino spectrum, neutrino interactions in the detector
array, the range and energy losses of the muon as well as the threshold energy for muons, has been determined by the Monte Carlo program WimpSim \cite{18}. In Tab. 5 the values of the calculated flux according to Eq. (5.9) are presented. These values are only upper flux limits, as the actual flux still is not known.

<table>
<thead>
<tr>
<th>Mass [GeV]</th>
<th>$\phi_\mu$ [km$^{-2}$a$^{-1}$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>$1.81 \times 10^9$</td>
</tr>
<tr>
<td>100</td>
<td>$2.30 \times 10^4$</td>
</tr>
<tr>
<td>250</td>
<td>$3.76 \times 10^3$</td>
</tr>
<tr>
<td>500</td>
<td>$1.24 \times 10^3$</td>
</tr>
<tr>
<td>1000</td>
<td>$6.05 \times 10^2$</td>
</tr>
<tr>
<td>3000</td>
<td>$3.70 \times 10^2$</td>
</tr>
<tr>
<td>5000</td>
<td>$3.48 \times 10^2$</td>
</tr>
</tbody>
</table>

Table 5: The values of the muon flux at the Earth, induced by neutrinos that have been produced through WIMP annihilations in the Sun’s center. This flux has been calculated to a 90% confidence level.

It is expected, that the muon flux equals the detectable neutrino flux, so the event rate is here obtained by the product of the flux from Tab. 5 with the geometric area of 1 km$^2$. Thus, only the unity in Tab. 5 will be changed, and therefore has not been listed up in this table. The values of the muon flux are only an upper limit to a 90% confidence level.
6 Conclusion

The results of high energy neutrino production in the Sun are summarized and compared to former calculations, as well as the flux by WIMP annihilations.

In the case of neutrinos that have been produced in interaction processes of cosmic rays with the Sun’s atmosphere and then been detected at IceCube, the analytically calculated event rate per year provides acceptable results. Electron neutrinos reach a value of $\dot{N}_{\nu_e} = 0.46 \, a^{-1}$ and muon neutrinos $\dot{N}_{\nu_\mu} = 2.12 \, a^{-1}$.

In Fig. 6 the event rates per year for both electron and muon neutrinos are shown as a function of the threshold energy. Especially for electron neutrinos a low threshold energy is important so that their event rate does not vanish. Muon neutrinos, on the other hand, have similar characteristics even though not as distinctive.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{event_rates.png}
\caption{Event rates for electron (red line) and muon neutrinos (blue line) as a function of the threshold energy $E_{\text{th}}$.}
\end{figure}

In former calculations the event rate was assumed to be about $\dot{N}_{\nu_e} = 24 - 46 \, a^{-1}$ and $\dot{N}_{\nu_\mu} = 46 - 82 \, a^{-1}$ with a detector volume of $V_{\text{det}} = 1 \, \text{km}^3$ [3].

A factor that crucially exerts influence on the discrepancy between the former results and those obtained in this paper is the effective area which is confirmed by Fig. 7. At lower and higher energies it can be seen that the former assumed effective area is about ten times higher as the current values, where lower energies influence the result much more (Fig. 6).
Further aspects are the squared mass differences as well as mixing angles that affect oscillation. In Ref. [3] their values are given as $\delta m^2_{12} = 1.9 \times 10^{-5}$ eV$^2$ and $|\delta m^2_{31}| = 3.5 \times 10^{-3}$ and accordingly $\Theta_{\text{sun}} = 0.62$ and $\Theta_{\text{atm}} = 1.03$ which both are quite different to those taken in this paper (see Tab. 4), too.

All the factors mentioned above explain why the detected muon neutrino event rates are about 20 times smaller than those expected in former calculations.

Comparing neutrino fluxes produced by cosmic ray interactions with the atmosphere of the Sun to those produced through WIMP annihilations, the latter is very much higher, which is also claimed in Ref. [3]. But considering, that this flux is not the actual one but only gives the upper limit to a confidence level of 90%, the calculated event rates cannot be interpreted as ones based on facts. It just lets assuming that it indeed exceeds the neutrino flux from the solar atmosphere.

Figure 7: Comparison of effective areas with former and current values. The blue line represents it of a 1 km$^3$ detector, and the gray line a $5 \times 10^{-3}$ km$^3$ detector, as calculated in the Appendix B. The dots represent IceCube’s current effective area for muon neutrinos (see Fig. 5).
A Appendix

This section displays all the antiderivatives that have been used to solve the integral in Eq. (5.1).

Electron neutrinos

The following approximating function has been found for electron neutrinos:

The antiderivatives are

\[
H_{\nu_e,1} = -\frac{2.209 \times 10^{-6}}{E^{2.03}} F_1(-2.03, 1.; -1.03; -8.5 \times 10^{-6}E) \\
+ \frac{1.293 \times 10^{-6}}{E^{1.98}} F_1(-1.98, 1.; -0.98; -8.5 \times 10^{-6}E)
\]  
(A.1)

\[
H_{\nu_e,2} = -\frac{2.714 \times 10^7}{E^{6.03}} F_1(-6.03, 1.; -5.03; -8.5 \times 10^{-6}E) \\
+ \frac{2.404 \times 10^7}{E^{5.98}} F_1(-5.98, 1.; -4.98; -8.5 \times 10^{-6}E) \\
+ \frac{13.279}{E^{4.03}} F_1(-4.03, 1.; -3.03; -8.5 \times 10^{-6}E) \\
- \frac{11.811}{E^{3.98}} F_1(-3.98, 1.; -2.98; -8.5 \times 10^{-6}E) \\
- \frac{3.645 \times 10^{-6}}{E^{2.03}} F_1(-2.03, 1.; -1.03; -8.5 \times 10^{-6}E)
\]  
(A.2)

\[
H_{\nu_e,3} = -\frac{1.747 \times 10^9}{E^{6.33}} F_1(-6.33, 1.; -5.33; -8.5 \times 10^{-6}E) \\
+ \frac{2.404 \times 10^7}{E^{5.98}} F_1(-5.98, 1.; -4.98; -8.5 \times 10^{-6}E) \\
+ \frac{835.068}{E^{4.33}} F_1(-4.33, 1.; -3.33; -8.5 \times 10^{-6}E) \\
- \frac{11.810}{E^{3.98}} F_1(-3.98, 1.; -2.98; -8.5 \times 10^{-6}E) \\
- \frac{2.146 \times 10^{-4}}{E^{2.33}} F_1(-2.33, 1.; -1.33; -8.5 \times 10^{-6}E)
\]  
(A.3)
\[ H_{\nu_e,4} = + \frac{8.842 \times 10^9 F_1(-6.38, 1.; -5.38; -8.5 \times 10^{-6}E)}{E^{6.38}} - \frac{1.747 \times 10^9 F_1(-6.33, 1.; -5.33; -8.5 \times 10^{-6}E)}{E^{6.33}} - \frac{4.210 \times 10^3 F_1(-4.38, 1.; -3.38; -8.5 \times 10^{-6}E)}{E^{4.38}} + \frac{835.068 F_1(-4.33, 1.; -3.33; -8.5 \times 10^{-6}E)}{E^{4.33}} - \frac{2.145 \times 10^{-4} F_1(-2.33, 1.; -1.33; -8.5 \times 10^{-6}E)}{E^{2.33}} \] (A.4)

With
\[
\begin{aligned}
H_{\nu_e,1}, & \quad E < 3860 \text{ GeV} \\
H_{\nu_e,2}, & \quad 3860 < E < 1.2 \times 10^6 \text{ GeV} \\
H_{\nu_e,3}, & \quad 1.2 \times 10^6 < E < 3.0 \times 10^6 \text{ GeV} \\
H_{\nu_e,5}, & \quad 3.0 \times 10^6 < E \text{ GeV}
\end{aligned}
\] (A.5)

**Muon neutrinos**

The following approximating function has been found for muon neutrinos:

The antiderivatives are

\[
\begin{aligned}
H_{\nu_\mu,1} &= - \frac{7.181 \times 10^{-7} F_1(-2.03, 1.; -1.03; -8.5 \times 10^{-6}E)}{E^{2.03}} - \frac{2.167 \times 10^{-6} F_1(-1.98, 1.; -0.98; -8.5 \times 10^{-6}E)}{E^{1.98}} \quad (A.6) \\
H_{\nu_\mu,2} &= + \frac{1.357 \times 10^{7} F_1(-6.03, 1.; -5.03; -8.5 \times 10^{-6}E)}{E^{6.03}} - \frac{6.640 F_1(-4.03, 1.; -3.03; -8.5 \times 10^{-6}E)}{E^{4.03}} - \frac{2.165 \times 10^{-6} F_1(-1.98, 1.; -0.98; -8.5 \times 10^{-6}E)}{E^{1.98}} \quad (A.7)
\end{aligned}
\]
\[ H_{\nu,3} = + \frac{1.357 \times 10^7 \, 2F_1(-6.03, 1; -5.03; -8.5 \times 10^{-6} E)}{E^{6.03}} \]
\[ - \frac{6.639 \, 2F_1(-4.03, 1; -3.03; -8.5 \times 10^{-6} E)}{E^{4.03}} \]
\[ + \frac{2.095 \times 10^4 \, 2F_1(-3.98, 1; -2.98; -8.5 \times 10^{-6} E)}{E^{3.98}} \]
\[ - \frac{6.566 \times 10^{-6} \, 2F_1(-1.98, 1; -0.98; -8.5 \times 10^{-6} E)}{E^{1.98}} \]
\[ - \frac{2.983 \times 10^{13} \, 2F_1(-5.98, 1; -4.98; -8.5 \times 10^{-6} E)}{E^{5.98}} \]  
\[ (A.8) \]

\[ H_{\nu,4} = + \frac{8.737 \times 10^8 \, 2F_1(-6.33, 1; -5.33; -8.5 \times 10^{-6} E)}{E^{6.33}} \]
\[ - \frac{4.175 \times 10^3 \, 2F_1(-4.33, 1; -3.33; -8.5 \times 10^{-6} E)}{E^{4.33}} \]
\[ + \frac{2.095 \times 10^4 \, 2F_1(-3.98, 1; -2.98; -8.5 \times 10^{-6} E)}{E^{3.98}} \]
\[ - \frac{6.566 \times 10^{-6} \, 2F_1(-1.98, 1; -0.98; -8.5 \times 10^{-6} E)}{E^{1.98}} \]
\[ - \frac{2.983 \times 10^{13} \, 2F_1(-5.98, 1; -4.98; -8.5 \times 10^{-6} E)}{E^{5.98}} \]  
\[ (A.9) \]

\[ H_{\nu,5} = - \frac{1.097 \times 10^{16} \, 2F_1(-6.38, 1; -5.38; -8.5 \times 10^{-6} E)}{E^{6.38}} \]
\[ + \frac{8.737 \times 10^8 \, 2F_1(-6.33, 1; -5.33; -8.5 \times 10^{-6} E)}{E^{6.33}} \]
\[ + \frac{7.469 \times 10^6 \, 2F_1(-4.38, 1; -3.38; -8.5 \times 10^{-6} E)}{E^{4.38}} \]
\[ - \frac{4.173 \times 10^2 \, 2F_1(-4.33, 1; -3.33; -8.5 \times 10^{-6} E)}{E^{4.33}} \]
\[ - \frac{2.142 \times 10^{-3} \, 2F_1(-2.38, 1; -1.38; -8.5 \times 10^{-6} E)}{E^{2.38}} \]  
\[ (A.10) \]

With
\[
\begin{aligned}
H_{\nu,1}, & \quad E < 3860 \text{ GeV} \\
H_{\nu,2}, & \quad 3860 < E < 8 \times 10^4 \text{ GeV} \\
H_{\nu,3}, & \quad 8 \times 10^4 < E < 1.2 \times 10^6 \text{ GeV} \\
H_{\nu,4}, & \quad 1.2 \times 10^6 < E < 3.0 \times 10^6 \text{ GeV} \\
H_{\nu,5}, & \quad 3.0 \times 10^6 < E \text{ GeV}
\end{aligned}
\]  
\[ (A.11) \]
In Ref. [3] the following integral has been solved in order to get a neutrino event rate per year

\[ \dot{N}_\nu = \int_{E_{th}}^{\infty} dE \, \phi_\nu \, \frac{\sigma_{CC}}{m_p} \frac{\rho}{A_{eff}} L_\nu(E) \, A, \]  

(B.1)

where \( \phi_\nu \) is the neutrino flux, \( \sigma_{CC} \) the neutrino nucleon cross sections, determined by the CTEQ4DIS parton distribution function [19], \( m_p \) the proton mass and \( A \) the detector area. In the case of muon neutrinos (see Conclusion) the factor \( L_\nu \) either is the lepton range or the detector thickness \( h \) depending on what parameter is larger; see also Eq. (3.14).

\[ L_\nu = \max \left\{ \frac{1}{\beta \rho} \ln \frac{E + \alpha/\beta}{E_{th} + \alpha/\beta}, h \right\} \]  

(B.2)

The effective area has then been calculated and is graphically displayed and compared to IceCube’s current one in Fig. 7.
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Mike Kroll
Dortmund, January 2012.
References


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